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Absorbing ideals of the form $I[[X]]$

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Abstract. Let R be a commutative ring with identity and n a positive integer. In [1], Anderson and Badawi define a proper ideal I of a commutative ring R to be n-absorbing if whenever $x_1...x_{n+1} \in I$ for $x_1,...,x_{n+1} \in R$, then there are n of the $x_i's$ whose product is in I. In this paper we investigate the transfer of the property n-absorbing from the ideal I of R to the ideal $I[[X]]$ of the formal power series ring $R[[X]]$.

Key Words: absorbing ideals, strongly absorbing ideals, formal power series rings.
2010 MSC: Primary 13A15; 13F25; 13F05; Secondary 13A99.

1 Introduction

All rings considered in this paper are commutative with an identity different from zero. Let R be a commutative ring and n be a positive integer. In [1], Anderson and Badawi define a proper ideal I of a commutative ring R to be n-absorbing if whenever $x_1...x_{n+1} \in I$ for $x_1,...,x_{n+1} \in R$, then there are n of the $x_i's$ whose product is in I. They also define $\omega_R(I) = \min\{n \mid I$ is an n-absorbing ideal of $R\}$. The ideal I is called strongly n-absorbing if whenever $I_1...I_{n+1} \in I$ for ideals $I_1,...,I_{n+1}$ of R, then there are n of the $I_i's$ whose product is in I. They define $\omega_R^*(I) = \min\{n \mid I$ is a strongly n-absorbing ideal of $R\}$. It is clear that if I is strongly n-absorbing, then it is n-absorbing, so $\omega_R(I) \leq \omega_R^*(I)$. They conjecture that the converse is true (Conjecture 1). It is clear that for $n = 1$, an ideal I is (strongly) 1-absorbing if and only if I is a prime ideal so Conjecture 1 is true for $n = 1$. Note that for $n = 2$, an ideal I of R is strongly 2-absorbing if and only if I is 2-absorbing [[4], Theorem 2.13]. Note also that in Prüfer domains the two concepts of n-absorbing and strongly n-absorbing ideals are equivalent. On the other hand, they conjecture that $\omega_{R[X]}(I[X]) = \omega_R(I)$ for any ideal I of R (Conjecture 3). A 1-absorbing ideal is just a prime ideal and it is well known that I is a prime ideal if and only if $I[X]$ is a prime ideal so Conjecture 3 is true for $n = 1$. In [1], the authors proved that Conjecture 3 is true for $n = 2$. Many authors investigated this conjecture. For example in [14], the author showed that Conjecture 3 is true if one of the following conditions hold:

(1) The ring R is a Prüfer domain.
(2) The ring R is a Gaussian ring such that its additive group is torsion free.
(3) The additive group of the ring R is torsion-free and I is a radical ideal of R.

In [13], the author proved that if I is a strongly n-absorbing ideal of R and R/I is Armendariz, then $I[X]$ is n-absorbing (R is said to be Armendariz, if $c(f)c(g) = 0$ for all $f,g \in R[X]$ such that $fg = 0$). Moreover, he proved that if I is n-absorbing, then $I[X]$ is n-absorbing in each of the following cases:

(1) The ring R/I is Armendariz and $|R/M| \geq n$ for each maximal ideal M of R containing I.
(2) The ring R/I is Armendariz and is $(n-1)!$-torsion-free as an additive group.
(3) The ring R/I is torsion-free as an additive group.

(4) The ring R/I is locally Bézout.

He showed also that Conjecture 3 is true in an arithmetical ring.

In this paper, we consider n-absorbing ideals of the form $I[[X]]$ of the power series ring $R[[X]]$. More precisely we explore the transfer of the property (strongly) n-absorbing from an ideal I of R to the ideal $I[[X]]$ of $R[[X]]$. The case $n = 1$ is clear since it is well known that an ideal I of R is prime if and only if the ideal $I[[X]]$ is prime. In [10], the authors proved that for an ideal I of a commutative ring R, I is 2-absorbing if and only if $I[[X]]$ is a 2-absorbing ideal of $R[[X]]$ (see also [13]). It was also shown in [10] that if R is a Prüfer domain, then I is n-absorbing if and only if $I[[X]]$ is n-absorbing. The proof was based on the characterization of absorbing ideals in Prüfer domains. In addition, they showed that if R is a Noetherian Gaussian u-ring, then I is n-absorbing if and only if $I[[X]]$ is n-absorbing (a commutative ring R is called u-ring provided R has the property that an ideal contained in a finite union of ideals must be contained in one of those ideals). Moreover, they proved that if R is a pseudo-valuation domain and I is an ideal of R with a non maximal radical, then $\omega_{R[[X]]}(I[[X]]) = \omega_R(I)$. On the other hand, in [14], the author proved that for a Dedekind domain R, $\omega_{R[[X]]}(I[[X]]) = \omega_R(I)$ for every ideal I of R. Moreover, if R is a Noetherian ring whose additive group is torsion-free, then $\omega_{R[[X]]}(I[[X]]) = \omega_R(I)$ for every radical ideal I of R.

In this paper we prove first that if the ideal $I[[X]]$ is n-absorbing, then the ideal I is strongly n-absorbing. Conversely, we prove that if the ideal I is strongly n-absorbing, then the ideal $I[[X]]$ is n-absorbing if one of the following conditions hold:

(1) The ring R is P-gaussian.

(2)The ring R is a Krull domain and I is a divisorial ideal.

(3) The ring R is a formally integrally closed domain and I is a t-ideal.

Most of the results proved here are based on content formulas for power series.

On the other hand, we prove that if the ideal I is n-absorbing, then $I[[X]]$ is n-absorbing if one of the following conditions hold:

(1) The ideal I is radical.

(2) The ring R is a Krull domain and I is of the form $(P_1...P_n)_v$ where the P_i are height one prime ideals of R.

(3) The ideal I has exactly n minimal prime ideals which are comaximal.

(4) The ideal I is a P-primary ideal where P is a prime ideal of R.

2 Absorbing ideals of the form $I[X]$

Let R be a commutative ring, n a positive integer and I a proper ideal of R. In [13], Laradji showed that if $I[X]$ is an n-absorbing ideal of $R[X]$, then I is a strongly n–absorbing ideal of R. We present here another proof which is completely different and which may be of independent interest, so we include it below.

Proposition 2.1. *Let R be a commutative ring, n a positive integer and I a proper ideal of R such that $I[X]$ is an n-absorbing ideal of $R[X]$ then I is a strongly n–absorbing ideal of R.*

Proof. By [[6], Lemma 2.1], let $I_1,...,I_{n+1}$ $(n+1)$ finitely generated ideals of R such that $I_1...I_{n+1} \subset I$. We shall prove that there are n of the $I_i^{\prime}s$ whose product is in I. For $j \in \{1,...,n+1\}$, put $I_j =< a_{1,j};...;a_{k_j,j} >$ and let $f_1 = a_{1,1}X + ... + a_{k_1,1}X^{k_1} \in I_1[X]$, $f_2 = a_{1,2}X^{k_1} + a_{2,2}X^{2k_1} + ... + a_{k_2,2}X^{k_1k_2} \in I_2[X]$,..., $f_{n+1} = a_{1,n+1}X^{k_1(k_2+1)...(k_n+1)} + ... + a_{k_{n+1},n+1}X^{k_1k_{n+1}(k_2+1)...(k_n+1)} \in I_{n+1}[X]$, then $f_1...f_{n+1} \in I_1[X]...I_{n+1}[X] \subset (I_1...I_{n+1})[X] \subset I[X]$. Hence there are n of the $f_i^{\prime}s$ whose product is in $I[X]$. Suppose for example that $f_1...f_n \in I[X]$, thus $a_{l_1,1}...a_{l_n,n} \in I$, for every $1 \leq l_i \leq k_i$ and $i \in \{1,...,n\}$. Hence, $I_1...I_n \subset I$. □

In the sequel, we will prove that for some class of rings, we have the equivalence: I is a strongly n-absorbing ideal of R if and only if $I[X]$ is an n-absorbing ideal of $R[X]$ and so $\omega_{R[X]}(I[X]) = \omega^*_R(I)$. Recall that a commutative ring R is called Gaussian if $c(fg) = c(f)c(g)$ for all $f, g \in R[X]$, where $c(f)$ denotes the content of the polynomial $f \in R[X]$.

Proposition 2.2. *Let R be Gaussian ring, n a positive integer and I a proper ideal of R. The ideal I is a strongly n-absorbing ideal of R if and only if $I[X]$ is an n-absorbing ideal of $R[X]$. Hence $\omega_{R[X]}(I[X]) = \omega^*_R(I)$.*

Proof. It is sufficient to prove that if I is strongly n-absorbing, then $I[X]$ is n-absorbing. Let $f_1, ..., f_{n+1} \in R[X]$ such that $f_1...f_{n+1} \in I[X]$ then $c(f_1...f_{n+1}) \subset I$. As R is a Gaussian ring then $c(f_1)...c(f_{n+1}) \subset I$. Since I is strongly n-absorbing, there are n of the $c(f_i)$'s whose product is contained in I. But $f_1...f_n \in c(f_1...f_n)[X] \subset c(f_1)...c(f_n)[X] \subset I[X]$. □

In [14], the author proved that if I is a radical n-absorbing ideal and the additive group of the ring R is torsion-free, then $I[X]$ is n-absorbing. In[11], the authors proved that if the ring R satisfies (**) (that is each proper ideal I of R with $\omega_R(I) < \infty$, $\omega_R(I) = |Min_R(I)|$, where $Min_R(I)$ denotes the set of prime ideals of R minimal over I), then if I is a radical n-absorbing ideal, then $I[X]$ is n-absorbing. Note that for a radical strongly n-absorbing ideal I, the ideal $I[X]$ is n-absorbing (without any additional assumption on the ring R) by the Dedekind-Mertens lemma. In the following proposition, we generalize the results of [14] and [11] by releasing the additional assumption on the ring R.

Proposition 2.3. *Let I be a proper radical ideal of a commutative ring R and n a positive integer. The following are equivalent:*

1. *I is a strongly n-absorbing ideal of R.*
2. *I is an n-absorbing ideal of R.*
3. *$I[X]$ is an n-absorbing ideal of $R[X]$.*
4. *$I[X]$ is a strongly n-absorbing ideal of $R[X]$.*
5. *$\forall k \in \mathbb{N}$, $I[X_1, ..., X_k]$ is an n-absorbing ideal of $R[X_1, ..., X_k]$.*
6. *$\forall k \in \mathbb{N}$, $I[X_1, ..., X_k]$ is a strongly n-absorbing ideal of $R[X_1, ..., X_k]$.*

Proof. $1 \Longrightarrow 2$ is clear.
$2 \Longrightarrow 3$ Since I is an n-absorbing ideal of R then $|Min_R(I)| \leq n$ by [[1], Theorem 2.5]. Let $P_1, ..., P_k$ the minimal prime ideals over I. Hence $I = \sqrt{I} = P_1 \cap ... \cap P_k$. Therefore $I[X] = P_1[X] \cap ... \cap P_k[X]$. By [[1], Theorem 2.1], $I[X]$ is k-absorbing so it is also n-absorbing.
$3 \Longrightarrow 1$ is clear.
The other equivalences result from the equality $\sqrt{I[X]} = \sqrt{I}[X]$, so since I is radical then $I[X]$ is also radical and then use an induction on $k \geq 1$.

□

Since every ideal of a von Neumann regular ring is radical, we get the following corollary:

Corollary 2.4. *Let R be a von Neumann regular ring, n a positive integer and I a proper ideal of R. The following are equivalent:*

1. *I is an n-absorbing ideal of R.*

2. *I is a strongly n-absorbing ideal of R.*
3. *$I[X]$ is an n-absorbing ideal of $R[X]$.*
4. *$I[X]$ is a strongly n-absorbing ideal of $R[X]$.*
5. *$\forall k \in \mathbb{N}$, $I[X_1,...,X_k]$ is an n-absorbing ideal of $R[X_1,...,X_k]$.*
6. *$\forall k \in \mathbb{N}$, $I[X_1,...,X_k]$ is a strongly n-absorbing ideal of $R[X_1,...,X_k]$.*

Recall that an ideal I of an integral domain R with quotient field K is called divisorial (or v-ideal) if $I = I_v$, where $I_v = (I^{-1})^{-1}$ and $I^{-1} = R : I = \{x \in K \mid xI \subset R\}$. In the sequel we prove that if I is a divisorial strongly n-absorbing ideal of an integrally closed domain R, then $I[X]$ is n-absorbing.

Lemma 2.5. *Let R be an integrally closed domain. For every $m \in \mathbb{N}^*$ and $f_1,...,f_m \in R[X]$, $(c(f_1...f_m))_v = (c(f_1)...c(f_m))_v$.*

Proof. By [[15], Lemme 1], if R is an integrally closed domain, then for every $f,g \in R[X]$, $(c(fg))_v = (c(f)c(g))_v$, hence the result is obtained by a simple induction on m. □

Proposition 2.6. *Let R be an integrally closed domain, I a divisorial ideal of R and n a positive integer. Then I is strongly n-absorbing if and only if $I[X]$ is n-absorbing. Hence $\omega_{R[X]}(I[X]) = \omega^*_R(I)$.*

Proof. Let $f_1,...,f_{n+1} \in R[X]$ such that $f_1...f_{n+1} \in I[X]$ then $c(f_1...f_{n+1}) \subset I$. Hence $(c(f_1...f_{n+1}))_v \subset I_v = I$. As R is integrally closed then $(c(f_1...f_{n+1}))_v = c(f_1)_v...c(f_{n+1})_v$. Therefore $c(f_1)...c(f_{n+1}) \subset I$. Since I is strongly n-absorbing then there are n of the $c(f_i)'s$ whose product is in I. Suppose for example that $c(f_1)...c(f_n) \subset I$. Consequently, $f_1...f_n \in c(f_1...f_n)[X] \subset c(f_1)...c(f_n)[X] \subset I[X]$. □

3 Absorbing ideals of the form $I[[X]]$

Let R be a commutative ring, I a proper ideal of R and n a positive integer. It is clear that if $I[[X]]$ is an n-absorbing ideal of $R[[X]]$, then $I[X]$ is an n-absorbing ideal of $R[X]$ and so I is a strongly n-absorbing ideal of R. In fact, let $f_1,...,f_{n+1} \in R[X]$ such that $f_1...f_{n+1} \in I[X]$ then $f_1...f_{n+1} \in I[[X]]$ so there are n of the $f_i's$ whose product is in $I[[X]] \cap R[X] = I[X]$.

Note that for a Noetherian ring R, if I is a strongly n-absorbing radical ideal, then $I[[X]]$ is an n-absorbing ideal. In fact, recall first that in [7], the authors established the following Dedekind-Mertens lemma for power series rings:

Proposition 3.1. *[7] Let R be a Noetherian ring and let $0 \neq g \in R[[X]]$. There exists a positive number k such that $c(f)^k c(g) = c(f)^{k-1}c(fg)$ for any $f \in R[[X]]$, where $c(f)$ is the ideal of R generated by the coefficients of f.*

Using this result, we prove that if I is a strongly n-absorbing radical ideal of a Noetherian ring R, then $I[[X]]$ is an n-absorbing ideal. Indeed, let $f_1,...,f_{n+1} \in R[[X]]$ such that $f_1...f_{n+1} \in I[[X]]$ then $c(f_1...f_{n+1}) \subset I$. By the Dedekind-Mertens lemma there exist positive integers $\alpha_1,..,\alpha_n$ such that $c(f_1)^{\alpha_1+1}c(f_2...f_{n+1}) = c(f_1)^{\alpha_1}c(f_1...f_{n+1}) \subset I$, $c(f_2)^{\alpha_2+1}c(f_3...f_{n+1}) = c(f_2)^{\alpha_2}c(f_2...f_{n+1}),...,c(f_n)^{\alpha_n+1}c(f_{n+1}) = c(f_n)^{\alpha_n}c(f_nf_{n+1})$. Now, we multiply the first equality by $c(f_2)^{\alpha_2}$, we get $c(f_1)^{\alpha_1+1}c(f_2)^{\alpha_2+1}c(f_3...f_{n+1}) \subset I$. Continuing this process, we get $c(f_1)^{\alpha_1+1}...c(f_n)^{\alpha_n+1}c(f_{n+1}) \subset I$. As I strongly n-absorbing then there exists $(k_1,...,k_{n+1}) \in \mathbb{N}^{n+1}$ such that $k_1 + ... + k_{n+1} = n$ and $c(f_1)^{k_1}...c(f_n)^{k_n}c(f_{n+1})^{k_{n+1}} \subset I$. Suppose for example that $k_{n+1} = 0$, so $c(f_1)^{k_1}...c(f_n)^{k_n} \subset I$. Since I is radical then $c(f_1)...c(f_n) \subset I$. But $f_1...f_n \in c(f_1...f_n)[[X]] \subset c(f_1)...c(f_n)[[X]] \subset I[[X]]$.

In the sequel we prove that the hypothesis R is Noetherian can be released. More precisely we show that if I is a radical n-absorbing ideal of a commutative ring R, then $I[[X]]$ is n-absorbing.

More generally, in the first part of this section, we prove that if I is an n-absorbing ideal of R, then $I[[X]]$ is an n-absorbing ideal of $R[[X]]$ if one of the following conditions hold:

1. The ideal I is radical.
2. The ring R is a Krull domain and I is of the form $(P_1...P_n)_v$ where the P_i are height one prime ideals of R.
3. The ideal I has exactly n minimal prime ideals which are comaximal.
4. The ideal I is a P-primary ideal where P is a prime ideal of R.

In the next proposition we generalize Corollary 16 of [14] for any commutative ring R.

Proposition 3.2. *Let I be a proper radical ideal of a commutative ring R and n a positive integer. The following are equivalent:*

1. *I is a strongly n-absorbing ideal of R.*
2. *I is an n-absorbing ideal of R.*
3. *$I[[X]]$ is an n-absorbing ideal of $R[[X]]$.*
4. *$I[[X]]$ is a strongly n-absorbing ideal of $R[[X]]$.*
5. *$\forall k \in \mathbb{N}$, $I[[X_1,...,X_k]]$ is an n-absorbing ideal of $R[[X_1,...,X_k]]$.*
6. *$\forall k \in \mathbb{N}$, $I[[X_1,...,X_k]]$ is a strongly n-absorbing ideal of $R[[X_1,...,X_k]]$.*

Proof. The proof is similar to the case of polynomial rings. For the sake of completeness, we include it here.
$1 \Longrightarrow 2$ is clear.
$2 \Longrightarrow 3$ Since I is an n-absorbing ideal of R then $|Min_R(I)| \leq n$ by [[1], Theorem 2.5]. Let $P_1,...,P_k$ the minimal prime ideals over I. Hence $I = \sqrt{I} = P_1 \cap ... \cap P_k$. Therefore $I[[X]] = P_1[[X]] \cap ... \cap P_k[[X]]$. By [[1], Theorem 2.1], $I[[X]]$ is k-absorbing so it is also n-absorbing.
$3 \Longrightarrow 1$ is clear.
The other equivalences result from the equality $\sqrt{I[[X]]} = \sqrt{I}[[X]]$. In fact, $\sqrt{I[[X]]} \subset \sqrt{I}[[X]]$ for any ideal I of R, since if P is a prime ideal of R containing I, then $P[[X]]$ is a prime ideal of $R[[X]]$ containing $I[[X]]$, so $\sqrt{I[[X]]} \subset P[[X]]$ for any prime ideal P containing I which implies that $\sqrt{I[[X]]} \subset \sqrt{I}[[X]]$. Conversely, if I is an n-absorbing ideal of R, then by [5], $(\sqrt{I})^n \subset I$ so $(\sqrt{I}[[X]])^n \subset (\sqrt{I})^n[[X]] \subset I[[X]]$, which implies that $\sqrt{I}[[X]] \subset \sqrt{I[[X]]}$ and then the equality $\sqrt{I[[X]]} = \sqrt{I}[[X]]$.
Now since I is radical then $I[[X]]$ is also radical and then use an induction on $k \geq 1$. □

Corollary 3.3. *Let R be a von Neumann regular ring, n a positive integer and I a proper ideal of R. The following are equivalent:*

1. *I is a strongly n-absorbing ideal of R.*
2. *I is an n-absorbing ideal of R.*
3. *$I[[X]]$ is an n-absorbing ideal of $R[[X]]$.*
4. *$I[[X]]$ is a strongly n-absorbing ideal of $R[[X]]$.*
5. *$\forall k \in \mathbb{N}$, $I[[X_1,...,X_k]]$ is an n-absorbing ideal of $R[[X_1,...,X_k]]$.*

6. $\forall k \in \mathbb{N}$, $I[[X_1,...,X_k]]$ *is a strongly* n*-absorbing ideal of* $R[[X_1,...,X_k]]$.

Proposition 3.4. *Let* R *be a Krull domain,* n *a positive integer and* $P_1,..,P_n$ *be heigt one prime ideals of* R *and* $I = (P_1...P_n)_v$ *then* $I[[X]]$ *is an* n*-absorbing ideal of* $R[[X]]$*. Hence* $\omega_{R[[X]]}(I[[X]]) = \omega_R(I)$.

Proof. Note that, by [[1], Corollary 4.5], the ideal I is n-absorbing. We have $I[[X]] = ((P_1...P_n)[[X]])_v = ((P_1...P_n).A[[X]])_v = ((P_1.A[[X]])...(P_n.A[[X]]))_v$. So $I[[X]] = ((P_1.A[[X]])_v...(P_n.A[[X]])_v)_v = ((P_1[[X]])_v...(P_n[[X]])_v)_v = (P_1[[X]]...P_n[[X]])_v$.
By [9], $R[[X]]$ is also a Krull domain and for each $k \in \{1,...,n\}$, $P_k[[X]]$ is a height one prime ideal of $R[[X]]$. Hence by [[1], Corollary 4.5], the ideal I is n-absorbing. □

In the following, we give two cases where the property n-absorbing is stable when passing from I to the ideal $I[[X]]$.

Proposition 3.5. *Let* I *be an* n*-absorbing ideal of a ring* R *such that* I *has exactly* n *minimal prime ideals which are comaximal then* $I[[X]]$ *is* n*-absorbing. Hence* $\omega_{R[[X]]}(I[[X]]) = \omega_R(I)$.

Proof. Let $\{P_1,...,P_n\}$ be the minimal prime ideals over I. By [[1], Corollary 2.15], $I = P_1...P_n = P_1 \cap ... \cap P_n$, so $P[[X]] = P_1[[X]] \cap ... \cap P_n[[X]]$. Again by Theorem 2.1 of [1], the ideal $I[[X]]$ is n-absorbing. □

Proposition 3.6. *Let* P *be a prime ideal of a ring* R *and* I *be a primary ideal of* R *such that* $P^n \subset I$ *then* $I[[X]]$ *is* n*-absorbing. Hence* $\omega_{R[[X]]}(I[[X]]) = \omega_R(I)$.
In particular if P^n *is a* P*-primary ideal of* R*, then* $P^n[[X]]$ *is* n*-absorbing. Moreover, if* M *is a maximal ideal of* R*, then* $M^n[[X]]$ *is* n*-absorbing.*

Proof. By [[1], Theorem 3.1], the ideal I is n-absorbing. By [[8], Corollary 4], $I[[X]]$ is a $P[[X]]$-primary ideal of $R[[X]]$ and $(P[[X]])^n \subset P^n[[X]] \subset I[[X]]$. So again by [[1], Theorem 3.1], the ideal $I[[X]]$ is n-absorbing. □

In the sequel, we prove that if I is a strongly n-absorbing ideal of R, then $I[[X]]$ is an n-absorbing ideal of $R[[X]]$ if one of the following conditions hold:

1. The ring R is P-Gaussian.
2. The ring R is a Krull domain and I is a divisorial ideal.
3. The ring R is a formally integrally closed domain and I is a t-ideal.

Recall from [16], that a commutative ring R is called P-Gaussian if for every $f, g \in R[[X]]$, $c(fg) = c(f)c(g)$. For example a Noetherian Gaussian ring is P-Gaussian.

Proposition 3.7. *Let* R *be a P-Gaussian ring,* n *a positive integer and* I *an ideal of* R*. Then* $I[[X]]$ *is* n*-absorbing if and only if* I *is strongly* n*-absorbing. Hence* $\omega_{R[[X]]}(I[[X]]) = \omega^*_R(I)$.

Proof. Let $f_1,...,f_{n+1} \in R[[X]]$ such that $f_1...f_{n+1} \in I[[X]]$ then $c(f_1...f_{n+1}) \subset I$. As R is a P-Gaussian ring then $c(f_1)...c(f_{n+1}) \subset I$. Since I is strongly n-absorbing then $c(f_1)...c(f_n) \subset I$ for example. But $f_1...f_n \in c(f_1...f_n)[[X]] \subset c(f_1)...c(f_n)[[X]] \subset I[[X]]$. □

Proposition 3.8. *Let* R *be an integral domain such that* $R = \bigcap_\alpha V_\alpha$ *where* $(V_\alpha)_\alpha$ *is a collection of rank one valuation overrings of* R *and* I *a strongly* n*-absorbing ideal such that* $I = \bigcap_\alpha IV_\alpha$ *then* $I[[X]]$ *is* n*-absorbing. In particular, if* R *is a Krull domain and* I *is a strongly* n*-absorbing divisorial ideal, then* $I[[X]]$ *is* n*-absorbing. Hence* $\omega_{R[[X]]}(I[[X]]) = \omega^*_R(I)$.

Proof. Consider the star operation $*$ defined by $E^* = \bigcap_{\alpha} IV_\alpha$, for every nonzero fractional ideal of R. By [[2], Theorem 2.5] for nonzero $f,g \in R[[X]]$, $(c(fg))^* = (c(f)c(g))^*$. Let $f_1,...,f_{n+1} \in R[[X]]$ such that $f_1...f_{n+1} \in I[[X]]$ then $c(f_1...f_{n+1}) \subset I$. Hence $c(f_1)...c(f_n) \subset (c(f_1)...c(f_n))^* = (c(f_1...f_n))^* \subset I^* = I$. Now the result follows from the fact that I is strongly n-absorbing. □

Now we can recover Corollary 11 of [14] since a Dedekind domain is a Krull domain in which every ideal is divisorial. Moreover a Dedekind domain is a Prüfer domain so by [[1], Corollary 6.9], every n-absorbing ideal is strongly n-absorbing.

Corollary 3.9. *Let R be a Dedekind domain, then $\omega_{R[[X]]}(I[[X]]) = \omega_R(I)$.*

More generally if R is a completely integrally closed domain and I is a strongly n-absorbing divisorial ideal, then $I[[X]]$ is n-absorbing by [[12], Theorem 2.11].

Recall from [3], that an integral domain R is called formally integrally closed if for nonzero $f,g \in R[[X]]$, $(c(fg))_t = (c(f)c(g))_t$, where $I_t = \cup\{J_v \mid J$ is a finitely generated non zero fractional ideal of R such that $J \subset I\}$, for every non zero fractional ideal I of R. A nonzero fractional ideal I of R is called a t-ideal if $I_t = I$. Integral domains R such that R_M is a one dimensional valuation domain for every t-maximal ideal of R are examples of formally integrally closed domains. We get then the following proposition:

Proposition 3.10. *Let R be a formally integrally closed domain, n a positive integer and I a strongly n-absorbing t-ideal then $I[[X]]$ is n-absorbing. Hence $\omega_{R[[X]]}(I[[X]]) = \omega^*_R(I)$.*

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