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S-strong Mori domain of $A + XB[X]$

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Abstract. Let A be an integral domain and S a multiplicative subset of A. We say that A is an S-strong Mori domain (S-SM domain) if for each nonzero ideal I of A there exist an $s \in S$ and a w-finite type ideal J of I such that $sI \subseteq J \subseteq I_w$ ([10]). Let $A \subseteq B$ be an extension of integral domains, S an anti-Archimedean multiplicative subset of A and X an indeterminate over B. In this note we give, with an additional assumption a necessary and sufficient condition for the polynomial ring $A + XB[X]$ to be an S-SM-domain.

Key Words: S-strong Mori domains, polynomial rings.
2010 MSC: Primary 13F05; 13E99. Secondary 13A15.

1 Introduction

Let A be an integral domain with quotient field K and let $\mathcal{F}(A)$ be the set of nonzero fractional ideals of A. For an $I \in \mathcal{F}(A)$, set $I^{-1} := \{x \in K \mid xI \subseteq A\}$. The mapping on $\mathcal{F}(A)$ defined by $I \mapsto I_v = (I^{-1})^{-1}$ is called the v-operation on A, the mapping on $\mathcal{F}(A)$ defined by $I \mapsto I_t := \bigcup\{J_v$, J is a nonzero finitely generated subideal of $I\}$ is called the t-operation on A and the mapping on $\mathcal{F}(A)$ defined by $I \mapsto I_w := \{x \in K \mid xJ \subseteq I$ for some finitely generated ideal J of A such that $J_v = A\}$ is called the w-operation on A. An $I \in \mathcal{F}(A)$ is a v-ideal (or divisorial ideal) (respectively, t-ideal, w-ideal) if $I_v = I$ (respectively, $I_t = I$, $I_w = I$). Recall that an ideal J of A is a Glaz-Vasconcelos ideal (GV-ideal) if J is finitely generated and $J^{-1} = A$. Let GV(A) be the set of GV-ideals of A. Then $I_w = \{x \in K \mid xJ \subseteq I$ for some $J \in GV(A)\}$ for all $I \in \mathcal{F}(A)$. An integral domain A is a Strong Mori domain (SM-domain) if A satisfies the ascending chain condition on integral w-ideals. Wang and McCasland have introduced the concept of SM-domains and investigated their properties [4], [5]. Recently, some more results were added by Park [12].

Let M be a (not necessarily torsion-free) module over an integral domain A and let $E(M)$ denote the injective envelope of M. Following [10], the w-closure of M is defined by $M_w = \{x \in E(M) \mid xJ \subseteq M$ for some $J \in$ GV$(A)\}$. We have $M \subseteq M_w \subseteq E(M)$. Note that by [9], if M is a torsion-free module, then the notion of w-closure coincide with the w-envelope of M defined by Fanggui and McCasland [4]. We say that M is a w-module if $M = M_w$, also M is w-finite type if M is a w-module and $M = B_w$ for some finitely generated submodule B of M. In [10], H. Kim, M. O. Kim and J. W. Lim generalized the notions of Strong Mori module and strong Mori domain by introducing the concept of S-strong Mori module (S-SM-module) and S-strong Mori domain (S-SM-domain). Let A be an integral domain, S a (not necessarily saturated) multiplicative subset of A and M a w-module as an A-module. We say that M is S-w-finite if $sM \subseteq F$ for some $s \in S$ and some w-finite type submodule F of M, and M is an S-strong Mori module (S-SM-module) if each w-submodule of M is S-w-finite. We say that a nonzero ideal I of A is S-w-finite if there exist an $s \in S$ and a w-finite type ideal J of I such that $sI \subseteq J \subseteq I_w$. We also define A to be an S-strong Mori domain (S-SM domain) if each nonzero ideal of A is S-w-finite. Recall that a multiplicative subset S of an integral domain A is said to be *anti-archimedean* if for each $s \in S$, $S \cap (\bigcap_{n\geq 1} s^n A) \neq \emptyset$ ([7]). Let $A \subseteq B$ be an extension of integral domains, S an anti-Archimedean multiplicative subset of A and X an indeterminate over B. In this note we give, with an additional assumption a necessary and sufficient condition for the polynomial ring $R = A + XB[X]$ to be an S-SM-domain. We show that if R is t-linked over $A[X]$, then the following conditions are equivalent.

1. R is an S-SM-domain.

2. A is an S-SM-domain and B is an S-w-finite module as A-module.

We also give a necessary and sufficient condition for the polynomial ring $R = A + XA_S[X]$ to be an SM-domain, where S is a multiplicative subset of A. We prove that R is an SM-domain if and only if A is an SM-domain and S consists of units of A.

2 Main results

Let A be an integral domain and S a multiplicative subset of A. We start this paper by giving a necessary and sufficient condition for the polynomial ring of the form $A + XA_S[X]$ to be an SM-domain.

Theorem 2.1. Let A be an integral domain, S a multiplicative subset of A and $R = A + XA_S[X]$. Then the following conditions are equivalent.

1. R is an SM-domain.

2. A is an SM-domain and S consists of units of A.

Proof. $(2) \Rightarrow (1)$. Obvious.

$(1) \Rightarrow (2)$. We will show that A is an SM-domain. Let I be an ideal of A. Since $I + XA_S[X]$ is an ideal of R, $I + XA_S[X]$ is a w-finite, thus there exist a $P_1, ..., P_n \in I + XA_S[X]$ such that $(I + XA_S[X])_w = (P_1R + \cdots + P_nR)_w$. We will prove that $I_{w_A} = (P_1(0)A + \cdots + P_n(0)A)_{w_A}$. Let a be an element of I. Then $aJ \subseteq P_1R + \cdots + P_nR$ for some $J \in \mathrm{GV}(R)$. Let $J = (f_1, ..., f_n)$. Note that for each $1 \leq i \leq n$, $af_i \in P_1R + \cdots + P_nR$. Then for each $1 \leq i \leq n$, $af_i(0) \in P_1(0)A + \cdots + P_n(0)A$ which implies that $aJ_0 \subseteq P_1(0)A + \cdots + P_n(0)A$, with $J_0 = f_1(0)A + \cdots + f_n(0)A$. Since $J \subseteq J_0R$, $R \subseteq J_0^{-1} \subseteq J_0^{-1}R \subseteq (J_0R)^{-1} \subseteq J^{-1} = R$; so $J_0^{-1} = A$. This implies that $J_0 \in \mathrm{GV}(A)$. As $aJ_0 \subseteq P_1(0)A + \cdots + P_n(0)A$ and $J_0 \in \mathrm{GV}(A)$, we obtain $a \in (P_1(0)A + \cdots + P_n(0)A)_{w_A}$. Moreover for each $1 \leq i \leq n$, $P_i(0) \in I$, hence $P_1(0)A + \cdots + P_n(0)A \subseteq I$.

Now, let $s \in S$. Assume that s is nonunit in A. We have $(\frac{1}{s^n}XR)_{n \in \mathbb{N}}$ is an increasing sequence of w-ideals of R. Since R is an SM-domain, the sequence $(\frac{1}{s^n}XR)_{n \in \mathbb{N}}$ is stationary; so there exits a positive integer n_0 such that $\frac{1}{s^{n_0}}XR = \frac{1}{s^{n_0+1}}XR$. This implies that $\frac{1}{s^{n_0}}XP = \frac{1}{s^{n_0+1}}XQ$ for some $P, Q \in R$. Thus $\frac{1}{s} \in A$, a contradiction. Whence we conclude the proof. □

Example 2.2. Let $\mathbb{Z}$ be the ring of integers, p a prime number and X an indeterminate over $\mathbb{Z}_{(p)}$. Then by the previous theorem, $R = \mathbb{Z} + X\mathbb{Z}_{(p)}[X]$ is not an SM-domain.

Let $A \subseteq B$ be an extension of integral domains. Following [2], we say that B is *t-linked over A*, if for each finitely generated ideal I of A with $I^{-1} = A$, we have $(IB)^{-1} = B$. For example, the polynomial ring $A[X]$ is t-linked over A. In general every flat extension is t-linked.

Proposition 2.3. *Let $A \subseteq B \subseteq C$ be extensions of integral domains. If C is t-linked over B and B is t-linked over A, then C is t-linked over A.*

Proof. Let I be a finitely generated ideal of A such that $I^{-1} = A$. We show that $(IC)^{-1} = C$. Since B is t-linked over A, $(IB)^{-1} = B$. It's easy to show that $(IB)C = IC$ is a finitely generated ideal of C. Moreover, as C is t-linked over B, then $(IC)^{-1} = ((IB)C)^{-1} = C$. Hence, C is t-linked over A. □

Corollary 2.4. *Let $A \subseteq B$ be an extension of integral domains and $R = A + XB[X]$. If R is t-linked over $A[X]$, then R is t-linked over A.*

Proof. Follows from the previous proposition and the fact that $A[X]$ is t-linked over A. □

Example 2.5. *Let $K \subseteq L$ be an extension of fields and $R = K + XL[X]$. Then R is t-linked over K. Indeed, by Corollary 2.4, it suffices to prove that R is t-linked over $K[X]$. But $GV(K[X]) = \{K[X]\}$ and $(K[X]R)^{-1} = R^{-1} = R$. Thus R is t-linked over $K[X]$; so R is t-linked over K.*

Let $A \subseteq B$ be an extension of integral domains and $R = A + XB[X]$. The following example proves that the extension $A[X] \subseteq R$ is not t-linked in general.

Example 2.6. *Let $A = \mathbb{Z}$ and $B = \mathbb{Z}_{(2)} = S^{-1}\mathbb{Z}$, with $S = \{2^n,\ n \in \mathbb{N}\}$ a multiplicative subset of $\mathbb{Z}$. Let $R = A + XB[X]$. We show that $R = \mathbb{Z} + X\mathbb{Z}_{(2)}[X]$ is not t-linked over $\mathbb{Z}[X]$. Let $I = 2\mathbb{Z} + X\mathbb{Z}[X] = (2, X)$. Then I is a finitely generated ideal of $\mathbb{Z}[X]$. Moreover, by [1, Lemma 2.1], $I^{-1} = \frac{1}{2}\mathbb{Z} \cap \mathbb{Z} + X\mathbb{Z}[X] = \mathbb{Z}[X]$. Suppose that R is t-linked over $\mathbb{Z}[X]$. Since I is a finitely generated ideal of $\mathbb{Z}[X]$, with $I^{-1} = \mathbb{Z}[X]$, then $(IR)^{-1} = R$. As $IR \subseteq 2\mathbb{Z} + X\mathbb{Z}_{(2)}[X]$, then $(2\mathbb{Z} + X\mathbb{Z}_{(2)}[X])^{-1} \subseteq (IR)^{-1} = R$. Again by [1, Lemma 2.1], $\frac{1}{2}\mathbb{Z} \cap \mathbb{Z}_{(2)} + X\mathbb{Z}_{(2)}[X] \subseteq R$, which implies that $\frac{1}{2} \in \mathbb{Z}$, a contradiction.*

Remark 2.7. Note that the extension $A[X] \subseteq R$ is not t-linked even if R is a flat A-module. Indeed, since $\mathbb{Z}_{(2)}$ is a flat $\mathbb{Z}$-module, then by [1, Lemma 3.6], $R = \mathbb{Z} + X\mathbb{Z}_{(2)}[X]$ is a flat $\mathbb{Z}$-module. But by the previous example, $R = \mathbb{Z} + X\mathbb{Z}_{(2)}[X]$ is not t-linked over $\mathbb{Z}[X]$.

Proposition 2.8. *Let $A \subseteq B$ be an extension of integral domains and $R = A + XB[X]$. If R is t-linked over $A[X]$, then B is t-linked over A.*

Proof. Let $I \in \mathrm{GV}(A)$. We show that $(IB)^{-1} = B$. By Corollary 2.4, R is t-linked over A, then $(IR)^{-1} = R$. Since $IR \subseteq I + XIB[X] \subseteq R$, then by [1, Lemma 2.1], $R \subseteq I^{-1} \cap (IB)^{-1} + X(IB)^{-1}[X] \subseteq (IR)^{-1} = R$. This implies that $B \subseteq (IB)^{-1} \subseteq B$, and hence $(IB)^{-1} = B$. □

Let A be an integral domain, S a (not necessarily saturated) multiplicative subset of A and M a w-module as an A-module. We say that M is *S-w-finite* if $sM \subseteq F$ for some $s \in S$ and some w-finite type submodule F of M, and M is an *S-strong Mori module (S-SM-module)* if each w-submodule of M is S-w-finite. We say that a nonzero ideal I of A is *S-w-finite* if there exist an $s \in S$ and a w-finite type ideal J of I such that $sI \subseteq J \subseteq I_w$. We also define A to be an *S-strong Mori domain* (S-SM domain) if each nonzero ideal of A is S-w-finite ([10]).

Recall that a multiplicative subset S of an integral domain A is said to be *anti-archimedean* if for each $s \in S$, $S \cap (\bigcap_{n \geq 1} s^n A) \neq \emptyset$. For example, if V is a valuation domain with no height-one prime ideals, then $V \setminus \{0\}$ is an anti-archimedean subset of V [3, Proposition 2.2]. Let A be an integral domain and S an anti-archimedean subset of A. According to [10], the authors gave a necessary and sufficient condition for the polynomial rings to be an S-SM-domain. They showed that A is an S-SM-domain if and only if $A[X]$ is an S-SM-domain ([10, Theorem 2.8]). Our next Theorem extend this result to the polynomial rings of the form $A + XB[X]$. First, note that by Proposition 2.8, if $R = A + XB[X]$ ($A \subseteq B$ is an extension of integral domains) is t-linked over $A[X]$, then B is t-linked over A. So B is a w-module as A-module.

Theorem 2.9. Let $A \subseteq B$ be an extension of integral domains, S an anti-Archimedean multiplicative subset of A and $R = A + XB[X]$. Assume that R is t-linked over $A[X]$. Then the following conditions are equivalent.

1. R is an S-SM-domain.

2. A is an S-SM-domain and B is an S-w-finite module as A-module.

Proof. (1) $\Rightarrow$ (2). First, we will show that A is an S-SM-domain. Let I be an ideal of A. Since $I+XB[X]$ is an ideal of R, $I+XB[X]$ is S-w-finite, thus there exist an $s \in S$ and a $P_1,\dots,P_n \in I+XB[X]$ such that $s(I+XB[X]) \subseteq (P_1R+\cdots+P_nR)_w \subseteq (I+XB[X])_w$. We will prove that $sI \subseteq (P_1(0)A+\cdots+P_n(0)A)_{w_A} \subseteq I_{w_A}$. Let x be an element of I. Since $sx \in s(I+XB[X])$, then $sxJ \subseteq P_1R+\cdots+P_nR$ for some $J \in \mathrm{GV}(R)$. Let $J=(f_1,\dots,f_n)$. Note that for each $1 \le i \le n$, $sxf_i \in P_1R+\cdots+P_nR$. Then for each $1 \le i \le n$, $sxf_i(0) \in P_1(0)A+\cdots+P_n(0)A$ which implies that $sxJ_0 \subseteq P_1(0)A+\cdots+P_n(0)A$, with $J_0 = f_1(0)A+\cdots+f_n(0)A$. Since $J \subseteq J_0R$, then $R \subseteq J_0^{-1} \subseteq J_0^{-1}R \subseteq (J_0R)^{-1} \subseteq J^{-1} = R$, thus $J_0^{-1} = A$. This implies that $J_0 \in \mathrm{GV}(A)$. As $sxJ_0 \subseteq P_1(0)A+\cdots+P_n(0)A$ and $J_0 \in \mathrm{GV}(A)$, then $sx \in (P_1(0)A+\cdots+P_n(0)A)_{w_A}$. Moreover for each $1 \le i \le n$, $P_i(0) \in I$, hence $(P_1(0)A+\cdots+P_n(0)A)_{w_A} \subseteq I_{w_A}$.

Now, we will show that B is an S-w-finite module (as A-module). By [1, Lemma 2.2], $XB[X]$ is an ideal divisorial of R; so $XB[X]$ is a w-ideal of R. Since R is an S-SM-domain, $s(XB[X]) \subseteq (XP_1R+\cdots+XP_nR)_w \subseteq (XB[X])_w = XB[X]$ for some $s \in S$ and some $P_1,\dots,P_n \in B[X]$. Let b be an element of B. Then there exists a finitely generated ideal J of R with $J^{-1}=R$ such that $sbXJ \subseteq XP_1R+\cdots+XP_nR$. Let $J=(f_1,\dots,f_n)$, where $f_1,\dots,f_n \in R$. Note that for each $1 \le i \le n$, $sbf_i \in P_1R+\cdots+P_nR$. Then for each $1 \le i \le n$, $sbf_i(0) \in P_1(0)A+\cdots+P_n(0)A$; so $sbJ_0 \subseteq P_1(0)A+\cdots+P_n(0)A$, where $J_0 = f_1(0)A+\cdots+f_n(0)A$. Since $J \subseteq J_0R$, then $J_0^{-1} \subseteq J_0^{-1}R \subseteq (J_0R)^{-1} \subseteq J^{-1} = R$, thus $J_0^{-1} = A$. This implies that $J_0 \in \mathrm{GV}(A)$. As $sbJ_0 \subseteq P_1(0)A+\cdots+P_n(0)A$ and $J_0 \in \mathrm{GV}(A)$, then $sb \in (P_1(0)A+\cdots+P_n(0)A)_{w_A}$. Moreover for each $1 \le i \le n$, $P_i(0) \in B$, then $(P_1(0)A+\cdots+P_n(0)A)_{w_A} \subseteq B_{w_A} = B$. Hence $sB \subseteq (P_1(0)A+\cdots+P_n(0)A)_{w_A} \subseteq B$.

(2) $\Rightarrow$ (1). Since B is an S-w-finite module, $sB \subseteq (a_1A+\cdots+a_nA)_{w_A} \subseteq B$ for some $s \in S$ and some $a_1,\dots,a_n \in B$. We will show that $sB[X] \subseteq (a_1A[X]+\cdots+a_nA[X])_{w_{A[X]}}$. Let $P = \sum_{i=0}^m b_iX^i \in B[X]$. Note that for each $0 \le i \le m$, $sb_iJ_i \subseteq a_1A+\cdots+a_nA$ for some $J_i \in \mathrm{GV}(A)$. Then for each $0 \le i,j \le m$, $sb_iX^jJ_i[X] \subseteq a_1A[X]+\cdots+a_nA[X]$. Since $A[X]$ is t-linked over A, $J_i[X] = J_iA[X] \in \mathrm{GV}(A[X])$. This implies that for each $0 \le i \le m$, $sb_iX^i \in (a_1A[X]+\cdots+a_nA[X])_{w_{A[X]}}$; so $sP \in (a_1A[X]+\cdots+a_nA[X])_{w_{A[X]}}$. Hence $sB[X] \subseteq (a_1A[X]+\cdots+a_nA[X])_{w_{A[X]}}$.

Now, we have

$$\begin{aligned} sR &\subseteq sA + sXB[X] \\ &\subseteq sA[X] + (a_1XA[X]+\cdots+a_nXA[X])_{w_{A[X]}} \\ &\subseteq (sA[X] + (a_1X)A[X]+\cdots+(a_nX)A[X])_{w_{A[X]}} \\ &\subseteq R_{w_{A[X]}}. \end{aligned}$$

Therefore R is an S-w-finite module as $A[X]$-module.

On the other hand, since A is an S-SM-domain and S an anti-Archimedean multiplicative subset of A, then $A[X]$ is an S-SM-domain. Thus by [10, Theorem 2.11(2)], R is an S-SM-module as $A[X]$-module. Now we will prove that R is an S-SM-domain. Let I be an ideal of R. Since I is a submodule of R (as $A[X]$-module), $sI \subseteq (P_1A[X]+\cdots+P_nA[X])_{w_{A[X]}} \subseteq I_{w_{A[X]}}$ for some $s \in S$ and a $P_1,\dots,P_n \in I$. Let $Q \in I$, we have $sQ \in (P_1A[X]+\cdots+P_nA[X])_{w_{A[X]}}$, then $sQJ \subseteq P_1A[X]+\cdots+P_nA[X]$ for some $J \in \mathrm{GV}(A[X])$. Put $L = JR$. As R is t-linked over $A[X]$, then $L \in \mathrm{GV}(R)$. Moreover, $sQL = sQ(JR) \subseteq P_1R+\cdots+P_nR$, thus $sQ \in (P_1R+\cdots+P_nR)_w$. This implies that $sI \subseteq (P_1R+\cdots+P_nR)_w \subseteq I_w$. Hence I is an S-w-finite ideal. □

In the particular case when $B = A$, we regain the result of Kim, O. Kim and Lim [10].

Corollary 2.10. *A is an S-SM-domain if and only if the polynomial ring $A[X]$ is an S-SM-domain.*

Corollary 2.11. *Let $A \subseteq B$ be an extension of integral domains and $R = A + XB[X]$. Assume that R is t-linked over $A[X]$. Then the following statements are equivalent.*

1. *R is an SM-domain.*
2. *A is an SM-domain and B is a w-finite type module as A-module.*

Example 2.12. Let $R = \mathbb{Q} + X\mathbb{R}[X]$. By Example 2.5, R is t-linked over $\mathbb{Q}[X]$. Note that by [11, Corollary 3.13], R is not an SM-domain. Thus by the proof of Theorem 2.9, R is not an SM-module as $\mathbb{Q}[X]$-module.

Let K be a field and A a subring of K. Using Theorem 2.9 in the case when S consists of units of A, we regain the result of Mimouni, ([11]) for the polynomial rings of the form $A + XK[X]$ to be an SM-domain.

Proposition 2.13. *Let K be a field, A a subring of K and $R = A + XK[X]$. Then the following conditions are equivalent.*

1. *R is an SM-domain.*
2. *$A = k$ is a field and $[K : k]$ is finite.*

Proof. (1) $\Rightarrow$ (2). Since R is an SM-domain, then R is a Mori domain; so by [6, Proposition 4.18], $A = k$ is a field. Moreover, by the proof of Theorem 2.9 ((1) $\Rightarrow$ (2)), $K = F_w$, for some finitely generated submodule F of K (as A-module). Since $A = k$ is a field, then $F_w = F$. This implies that $[K : k]$ is finite.

(2) $\Rightarrow$ (1). If A is a field, then A is an SM-domain. Moreover since $[K : A]$ is finite, then K is an A-module of finite type, which implies that K is a w-finite type. Now, by Example 2.5, R is t-linked over $A[X]$. Thus by Theorem 2.9, R is an SM-domain. □

Question. Is the hypothesis $R = A + XB[X]$ is t-linked over $A[X]$ in Theorem 2.9 is necessary?

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References

[1] D.F. Anderson, S.E. Baghdadi and S. E. Kabbaj. On the class group of $A + XB[X]$ domains. Advances in Commutaive Ring Theory. Lecture Notes in Pure and Appl. Math. Marcel Dekker, 205 (1999), 73-85.

[2] D.D. Anderson, E.G. Houston and M. Zafrullah, t-linked extensions, the t-class group, and Nagat's theorem, J. Pure Appl. Algebra, 86 (1993), 109-129.

[3] D.D. Anderson, B.G. Kang and M.H. Park, *Anti-archimedean rings and power series rings*, Commun. Algebra, 26 (1998), 3223-3238.

[4] W. Fanggui and R.L. McCasland, On w-modules over Strong Mori domains, Commun. Algebra, 25 (1997), 1285-1306.

[5] W. Fanggui and R.L. McCasland, On Strong Mori domains, J. Pure Appl. Algebra, 135 (1999), 155-165.

[6] S. Gabelli and E. Houston, Coherent-like conditions in pullbacks, Michigan Math. J., 44 (1997), 99-122.

[7] A. Hamed and S. Hizem, S-Noetherian rings of the forms $\mathcal{A}[X]$ and $\mathcal{A}[[X]]$, Commun. Algebra, 43 (2015), 3848-3856.

[8] A. Hamed and S. Hizem, Modules satisfying the S-Noetherian property and S-ACCR, Commun. Algebra, 44 (2016), 1941-1951.

[9] H. Kim, Module-theoretic characterizations of t-linkative domains, Commun. Algebra, 36 (2008), 1649ÂŰ1670.

[10] H. Kim, M.O. Kim and J.W. Lim, On S-strong Mori Domains, J. Algebra, 416 (2014), 314-332.

[11] A. Mimouni, TW-domains and strong Mori domains, J. Pure Appl. Algebra, 177 (2003), 79-93.

[12] M.H. Park, Group rings and semigroup rings over Strong Mori domains, J. Pure Appl. Algebra, 163 (2001), 301-318.